

Submicron probes for Hall magnetometry over the extended temperature range from helium to room temperature

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We report on mesoscopic Hall sensors made from various materials and their suitability for accurate magnetization studies of submicron samples over a wide temperature range and, especially, at room temperature. Among the studied devices, the best stability and sensitivity have been found for Hall probes made from a high-concentration two-dimensional electron gas (HC-2DEG). Even at 300 K, such submicron probes can reliably resolve local changes in dc magnetic field of ≈ 1 G, which corresponds to a flux sensitivity of less than $0.1 \phi_0$ ($\phi_0 = h/e$ is the flux quantum). The resolution increases 100 times at temperatures below 80 K. It is also much higher for the detection of ac magnetic fields because resistance fluctuations limiting the low-frequency stability of the studied devices can be eliminated. Our second choice for room-temperature Hall micromagnetometry is gold Hall probes, which can show a sensitivity of the order of 10 G. The capabilities of HC-2DEG and gold micromagnetometers are demonstrated by measuring nm-scale movements of individual domain walls in a ferromagnet. © 2003 American Institute of Physics. [DOI: 10.1063/1.1576492]

I. INTRODUCTION

Mesoscopic Hall probes made from a two-dimensional electron gas (2DEG) have proved themselves as a valuable experimental tool for studies of magnetic flux distribution in macroscopic^{1,2} and submicron^{3,4} superconductors and for studies of the magnetic properties of individual nanometer-sized magnets and their arrays.^{5–9} This relatively simple technique, generally referred to as Hall micromagnetometry, exhibits remarkable sensitivity at low temperatures, allowing measurements of magnetic fields induced by mesoscopic objects at the level of 10^{-2} G/ $\sqrt{\text{Hz}}$ (for the case of dc signals) and 10^{-4} G/ $\sqrt{\text{Hz}}$ for ac signals.^{2–8} For a Hall cross of $1 \mu\text{m}$ in size, this corresponds to a flux resolution of $\approx 10^{-3}$ (10^{-5}) ϕ_0 and, in terms of magnetization, allows the detection of magnetic moments as small as 10^5 (10^3) μ_B for dc and ac measurements, respectively.^{2,3,6,8}

At low temperatures, the miniature 2DEG probes are widely used for studies of mesoscopic phenomena where they provide a viable alternative to micron-sized superconducting quantum interference devices.¹⁰ Generally, the operational range of 2DEG Hall μ -sensors is not limited to low temperatures⁹ but their sensitivity rapidly deteriorates at temperatures above 100 K,^{2,3,6,9,11} mainly because of a rapid increase in low-frequency resistance fluctuations. At the same time, many research areas require and would benefit from μ

probes suitable for magnetization measurements at higher temperatures. Such an extension of the operational range of Hall micromagnetometry to room temperature is particularly important for research on nanomagnetism and magnetic materials, as well as for possible applications in life sciences. With these applications in mind, we have fabricated and tested Hall μ sensors made from a variety of materials (namely, thin films of Bi, Al, Au, and Nb, epitaxial and δ -doped layers of GaAs and InSb, and a number of 2D systems based on GaAs/GaAlAs heterostructures). In this article, we describe our experience with these devices, concentrating on the operation of Hall probes found to be most suitable for room-temperature micromagnetometry.

II. EXPERIMENTAL DEVICES AND MEASUREMENTS

Examples of our experimental structures are shown in Fig. 1. These Hall probes were microfabricated by electron-beam lithography followed by thermal evaporation and lift-off (in the case of metal films) and by wet etching (in the case of semiconducting structures). The measurements were carried out using the standard low-frequency (30–1000 Hz) lock-in technique with an integration time of 0.3–3 s (Stanford Research lock-in amplifier model 830). For Hall sensors made from metal films, it was essential to use transformer preamplifiers (Stanford Research preamplifier model 554) to match the low input resistance of the measurement circuit. ac driving currents I for the semiconducting and metal sensors were of the order of 10 μA and 10 mA, respectively. An

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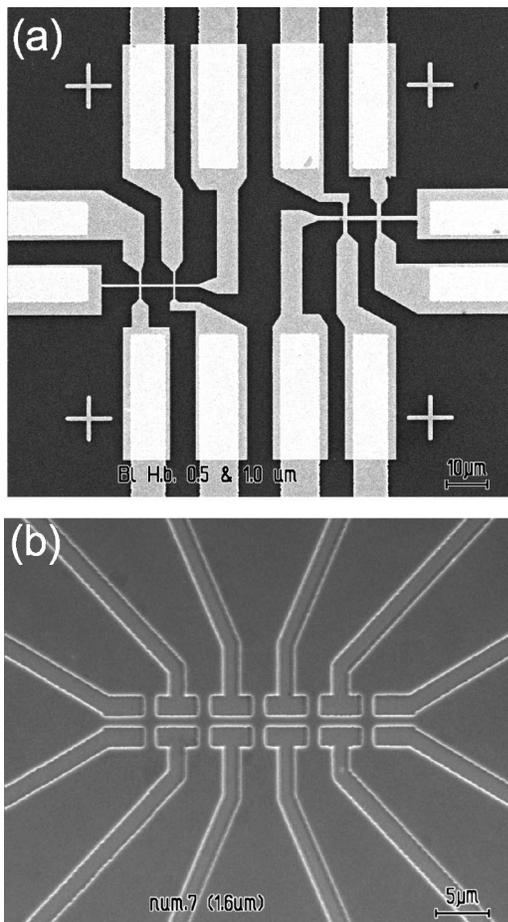


FIG. 1. Examples of the studied mesoscopic Hall devices. (a) Scanning electron micrograph showing two sets with 2 Hall crosses each made from Bi and having widths of 0.5 and 1 μm . (b) 2DEG Hall probes: the micrograph shows a mesa with five crosses of equal size wet-etched in a GaAs–AlGaAs heterostructure. Here, the nominal width of the crosses w (defined by lithography) is 1.6 μm . We have used 2DEG probes with sizes down to 0.5 μm .

optimal current I_o was carefully selected (within a factor of 2) for each individual Hall cross by measuring its performance over a wide range of I . At low currents $I \ll I_o$, the sensitivity was limited by voltage noise (Johnson noise: $V = \sqrt{4kRT}$, where kT is the thermal energy and R is the series resistance of the measurement circuit). The use of higher driving currents ($I \approx I_o$) has allowed us to suppress the actual contribution of the Johnson noise to the measured resistance (note that Johnson noise is independent of I while the generated Hall voltage increases linearly with I). However, we have found that above a certain current $I > I_o$ the signal-to-noise ratio cannot be improved any further for several reasons. The most important of them is the presence of slow resistance fluctuations. These fluctuations exhibit a $1/f$ -type behavior (see Figs. 2 and 3) and, at $I \approx I_o$, exceed the contribution from the Johnson noise usually by a factor of 10–1000. Furthermore, in the case of 2DEG devices, high currents $I > I_o$ can also lead to additional resistance instabilities.

III. OVERVIEW OF EXPERIMENTAL RESULTS

Figure 2 summarizes our experience with various mesoscopic Hall devices. It shows a typical Hall response of sev-

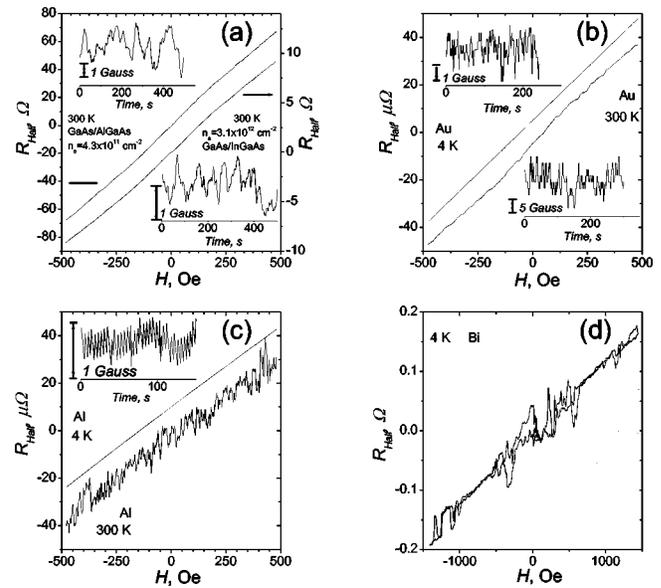


FIG. 2. Hall response R_{Hall} of various submicron probes. The insets show noise in the measured signals vs time (some of the curves were taken while sweeping H). All the measurements were carried out at frequency $f = 30.5$ Hz with time constant $\tau = 3$ s. Note that the high-frequency noise component seen in insets (b) and (c) is due to a finite digital resolution of lock-ins (the measured Hall signal increases with increasing H by minor steps due to digitalization). If necessary, this artifact can be eliminated by compensating a relatively large zero-field offset present in some devices. To compare the field resolution of different devices, the y scale for the noise signals is recalculated from ohms into gauss, using the measured Hall coefficients. (a) R_{Hall} at 300 K for crosses made from standard and high-concentration 2DEGs. The devices' geometry is shown in Fig. 1(b), the width $w \approx 2$ μm . (b) R_{Hall} for an Au Hall cross ($w \approx 0.6$ μm) at helium and room temperatures. The film thickness d is ≈ 100 nm. (c) Behavior of an Al Hall sensor at 4 and 300 K ($w \approx 0.4$ μm , $d \approx 50$ nm). (d) R_{Hall} for a Bi Hall cross with $w \approx 0.8$ μm and $d \approx 100$ nm. The large resistance fluctuations dominating the curve are irreproducible but tend to occur in certain field intervals. Even for the quieter parts of the curve, the resistance noise limits the field resolution of the Bi sensors to several tens of gauss. This curve shows that, despite the much lower concentration of carriers in Bi compared to Au or Al and, accordingly, the four orders of magnitude larger Hall response, Bi Hall sensors provide a much poorer resolution in terms of magnetic field.

eral of them to perpendicular magnetic field H swept over a relatively large field interval. The insets show the corresponding noise in R_{Hall} (recalculated in terms of measured magnetic field B). At room temperature, the best signal-to-noise ratio among the tested semiconducting devices has been found for Hall probes made from a high-concentration (HC)-2DEG (electron concentration $n > 10^{12}$ cm^{-2}) [see Fig. 2(a)]. Here, random resistance fluctuations (at the optimal current I_o) lead to a noise signal that corresponds to field changes of less than ≈ 1 G. This noise is dominated by very slow fluctuations (with a characteristic period > 100 s comparable with time of typical measurements). Figure 3 shows the corresponding low-frequency noise spectrum. For devices with lower n but of similar size and geometry, the Hall signal increases (as $1/n$) but so do resistance fluctuations, resulting in somewhat lower sensitivity [Fig. 2(a)]. All our micron-sized probes made from semiconductors with n larger than, say, $\approx 3 \times 10^{11}$ cm^{-2} were operational at 300 K and could detect changes on the level of 1–10 G. The best performance over the temperature range from 100 to 300 K

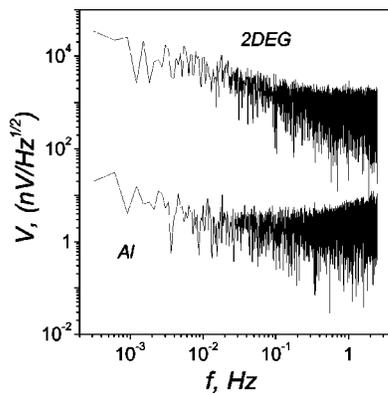


FIG. 3. The measured $1/f$ noise for Hall devices made from a HC-2DEG and Al. The measurements were carried out in the Hall geometry at room temperature and in the dark by using $50 \mu\text{A}$ current. Our Au devices exhibited behavior similar to Al probes but $1/f$ noise was several times smaller.

was observed for sensors made from a molecular beam epitaxy-grown HC-2DEG,¹² due to its relatively high mobility and lower series resistances involved. At temperatures below 80 K, their sensitivity typically increased to $\approx 10^{-2}$ G. The latter regime is well documented in literature²⁻⁷ and, therefore, not discussed below.

As concerns metal films, they are generally considered to be a poor choice for making Hall probes because of their high carrier concentration and, hence, very small Hall constants. This argument somehow appears to be not true for the case of mesoscopic Hall devices at room temperature. As one can see from Fig. 2(b), Au sensors show a million times smaller Hall response but, in terms of magnetic field B , their signal-to-noise ratio is comparable to the one exhibited by the 2DEG devices. The mesoscopic Au devices exhibit the sensitivity on the level of several gauss or $\approx \phi_0$ over the whole temperature range [see Fig. 2(b) and the next section].

All the other submicron probes made from metal films and tested in our experiments have shown notably larger resistance noise and lower sensitivity to dc magnetic fields. As an example, Fig. 2(c) shows a Hall response of Al probes: the field resolution is only ≈ 100 G at room temperature. It increases dramatically (to less than 1 G) at helium temperatures (exceeding the sensitivity of our Au probes) but still it is two to three orders of magnitude worse than the sensitivity of 2DEG probes at low temperatures.³

Submicron devices made from Bi present an interesting and nontrivial case. Due to its very low carrier concentration and large Hall response, Bi films continue to be viewed by many researchers as the material of choice for making small Hall sensors. However, in our experience, submicron Bi devices have always shown the worst performance, even at 4 K [Fig. 1(d)]. Random resistance fluctuations and telegraph noise obscure the Hall curves completely, making such Bi devices impractical for magnetization measurements. In addition, while both semiconducting and other metal devices have proved to be fairly robust in operation, did not require any special precautions, and could survive many cool-downs and measurements, our submicron Bi sensors were found to be prone to easy electrical damage for reasons that remain unclear to us. We note, however, that if Bi devices are pre-

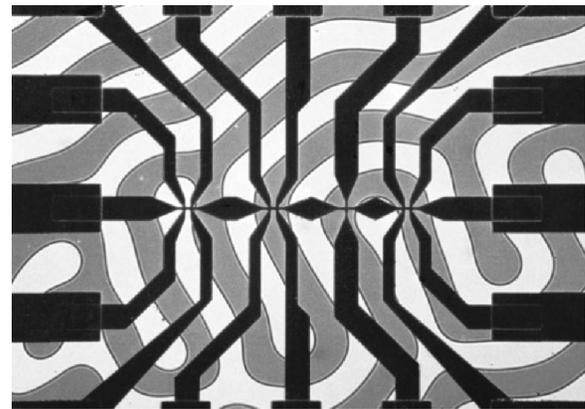


FIG. 4. Mesoscopic Au sensors microfabricated directly on top of an yttrium-iron garnet film. Magnetic domains in the garnet are clearly visible on the photograph which is taken in transmitted polarized light using a high-resolution optical microscope (domain width at 300 K is $\approx 14 \mu\text{m}$). Dark areas are the gold film.

pared by other methods (e.g., using epitaxial growth), it is still possible that the problems we experienced can be eliminated or would become less severe.

We should also mention that the discussed dc resolution of metallic Hall sensors (down to ≈ 1 G) can only be achieved in applications where relatively large ac magnetic fields (> 1 G) do not influence measurements (e.g., do not change magnetization of a studied object). Such ac fields are induced by high driving currents (10 mA per μm of width) required for the metal sensors to suppress the Johnson noise. On the other hand, a very important advantage of metal probes (and Au probes, in particular) is that they can be microfabricated directly on top of a sample of interest, which is not possible in the case of 2DEG probes and could be crucial for many experiments. Furthermore, metallic Hall probes can be made even smaller than 100 nm while this is practically impossible for semiconducting devices because of the presence of a depletion region.

IV. APPLICATION OF MESOSCOPIC HALL SENSORS FOR DETECTION OF MOVEMENTS OF FERROMAGNETIC DOMAIN WALLS

In order to demonstrate the operation of the described Au and HC-2DEG Hall sensors in a real experiment and give more details about their operation, we describe below their application for the detection of mesoscopic movements of individual domain walls in a ferromagnet. Figure 4 shows one of our Au devices placed on top of a garnet film. The photograph is taken in transmitted polarized light and allows one to see a magnetic domain structure underneath the Hall probes. The Au film is 50 nm thick and has been evaporated directly on the insulating garnet film. Hall crosses have different widths w ranging between 100 nm and $2 \mu\text{m}$. Their two-probe resistance R is about 20Ω at room temperature, decreasing by a factor of 2 at helium temperatures.

The geometry of our 2DEG sensors used in this application is similar to the one shown in Fig. 1(b). They have been fabricated from a specially designed InGaAs-AlGaAs-GaAs heterostructure¹² with a HC-2DEG embedded 50 nm

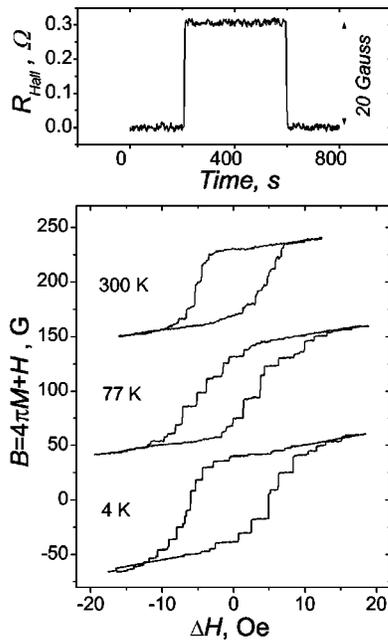


FIG. 5. Local magnetic field B measured by HC-2DEG probes as a domain wall creeps underneath a micron-sized Hall cross. The external magnetic field H is slowly swept up and down, forcing domain walls to move. For clarity, curves at different temperatures are shifted by 100 G, and $\Delta H = 0$ is chosen to be approximately at the center of the hysteresis loops. Top panel: changes in the Hall signal at 300 K after the perpendicular external field of 20 G is applied in the absence of the garnet film. Here, one can see that the noise level, which cannot be resolved on the scale of the main figure, is about 1 G.

below the surface. The 2DEG has $n \approx 4 \times 10^{12} \text{ cm}^{-2}$ and a very high room-temperature mobility of $0.8 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ (but increasing only to $2.6 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ at 4 K). The devices have $R < 10 \text{ k}\Omega$ at 300 K. A mm-sized piece of a garnet film has been placed in firm mechanical contact with the top surface and then fixed by a vacuum grease. Quantitative analysis of the shapes of the measured magnetization curves in Figs. 5 and 6 shows¹³ that this procedure allows us to achieve the separation between the garnet film and a 2DEG of less than $0.2 \mu\text{m}$.

The used yttrium-iron garnet film is $20 \mu\text{m}$ thick and has its magnetization in the direction perpendicular to the surface. The saturation magnetization is $\approx 200 \text{ G}$. The domain width is $\approx 14 \mu\text{m}$, and the width of domain walls is estimated to be $\approx 100 \text{ nm}$ at 300 K, decreasing to $\approx 15 \text{ nm}$ at 4 K. In our measurements, we have applied a perpendicular field H , forcing domains of the parallel polarity to grow at the expense of domains with the opposite polarity. As one of the domain walls reaches the central sensitive area of the probe, the measured Hall signal starts reversing its sign (at room temperature this process was simultaneously monitored in an optical microscope). Figures 5 and 6 plot changes in the local field B caused by a domain wall moving (creeping) over the Hall cross in forward and backward directions. The observed hysteresis is due to pinning on local defects and the steps correspond to jumps of domain walls from one pinning site to another (so-called Barkhausen noise but now it is measured for a single domain wall). One can see that the hysteresis loops become wider with decreasing temperature,

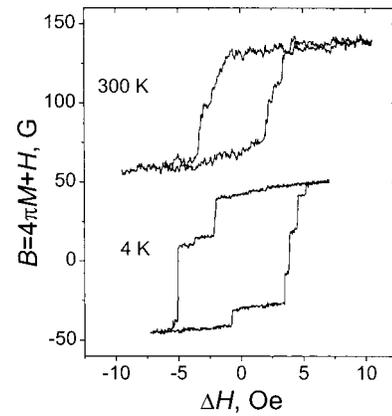


FIG. 6. Local movements of a ferromagnetic domain wall monitored by an Au Hall cross ($w \approx 0.6 \mu\text{m}$, $d \approx 50 \text{ nm}$). Labeling and procedures as in Fig. 5.

which indicates an increase in pinning. The smallest jump we could resolve at 300 K using HC-2DEG sensors corresponds to an average shift of an individual domain wall by only 30 nm, i.e., much smaller than the width of the domain wall itself. In the case of Au probes, the jumps at 300 K are poorly resolved because of large resistance noise and, also, due to smearing of the steps by the ac field induced by the driving current. In the experiment in Fig 6, ac fields were $\approx 10 \text{ G}$ and could de-pin domain walls in the garnet film. We note, however, that these garnets have shown very low pinning of domain walls¹³ and, for magnetic systems with higher coercivity, one should be able to increase the resolution of Au μ sensors further by using higher currents. Further details of the observed behavior of ferromagnetic domain walls ($< 80 \text{ K}$) are given elsewhere.¹³

V. DISCUSSION

The sensitivity of Hall sensors is fundamentally limited by the Johnson noise $V = \sqrt{4kRT}$. In the case of our Au sensors with Hall resistivity $\rho_{xy} \approx 0.1 \mu\Omega/\text{G}$, $I_o \approx 10 \text{ mA}$, and $R \approx 100 \Omega$, this noise limits the field resolution to $\approx 1 \text{ G}/\sqrt{\text{Hz}}$ at 300 K and $0.1 \text{ G}/\sqrt{\text{Hz}}$ at 4 K. In practice, however, we have always encountered additional low-frequency fluctuations in resistance ($1/f$ noise), as discussed above. For the case of Au probes, these fluctuations usually exceeded the Johnson noise by a factor of 10 (at $I = I_o$) and reduced the field resolution accordingly.

Non-Johnson noise is even more important for the case of our semiconducting devices, reducing their field resolution at all temperatures by a huge factor of 100 to 1000. This noise behaves as $1/f$ at frequencies below $\approx 1000 \text{ Hz}$ (see, e.g., Ref. 12) and did not show any saturation down to 0.001 Hz (Fig. 3). Moreover, the dc field resolution of semiconducting Hall devices depends crucially on their width w . For a $70 \mu\text{m}$ cross made from a HC-2DEG, we have reached the noise level of 10^{-3} G at 300 K (Ref. 12) but crosses smaller than $2 \mu\text{m}$ (with only slightly higher two-probe resistance) become increasingly noisier (yielding the dc resolution of $\approx 1 \text{ G}$ as shown in Figs. 2 and 5). For $w \leq 0.5 \mu\text{m}$, we found them no longer superior to Au μ probes for room-temperature applications.

Previously, it was suggested that it is DX centers that are responsible for the resistance noise and limit the high-resolution regime of 2DEG Hall probes to low temperatures.^{6,14} However, the strong dependence of the amplitude of the resistance fluctuations on the size w of 2DEG devices may indicate that there is also another mechanism for the noise at elevated temperatures. This additional noise could originate from the small number of electrons in the sensitive area of a 2D device. Indeed, for a standard 2DEG with $n \approx 3 \times 10^{11} \text{ cm}^{-2}$, there are only $N = 3000$ electrons in a $1 \mu\text{m}$ cross. At temperatures above the Fermi energy $E_F \approx 100 \text{ K}$, the 2DEG becomes classical and the number of electrons in the Hall cross should fluctuate. This means that all transport characteristics should exhibit thermodynamic fluctuations due to the number fluctuations (note that both ρ_{xx} and $\rho_{xy} \propto 1/n$). The discussed noise should be proportional to the driving current and thus can experimentally be distinguished from the Johnson noise. Unfortunately, we are not aware of any theory which would address the classical noise in open systems with a small number of electrons inside. Furthermore, one cannot use the known statistical theory¹⁵ for a gas of neutral particles (where the number fluctuations are given by \sqrt{N}), as the corresponding formulas are not applicable in our case because of strong screening.

It is also worth mentioning that—contrary to the common opinion—a lower carrier concentration does not necessarily lead to higher sensitivity of small Hall devices. Indeed, although ρ_{xy} decreases as $1/n$ with increasing n , high-concentration devices can also sustain higher currents (in our experience, $I_o \propto n$) and, hence, induced Hall voltages do not necessarily decrease. On the other hand, noise is generally expected to become smaller for better conductive high-concentration devices, which results in their better signal-to-noise ratio. This argument is also consistent with our observation that mesoscopic Au Hall devices exhibit much better characteristics than similar ones made from Bi, despite a much lower concentration of carriers in Bi.

VI. CONCLUSION

We have demonstrated that mesoscopic probes made from a high-concentration 2DEG and Au allow accurate micromagnetization measurements over the whole temperature range below room temperature and, in particular, are suitable for the detection of microscopic movements of ferromagnetic domain walls. The most unexpected and potentially useful result of our investigation is that, at high temperatures, sub-

micron Hall devices made from ordinary metals can exhibit a sensitivity to local dc magnetic fields comparable to the sensitivity of semiconducting devices. Among the tested metallic probes, the most sensible alternative to the 2DEG sensors was found in submicron Hall probes made from gold.

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